NETWORKS IN TRANSPORTATION – THEORY

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I. Introduction

Networks have always been important in transportation and telecommunication. However, they have become more important for all businesses today, especially because of the Internet. The Internet (originally conceived as a "network of networks") has connected virtually everything today. It has connected everybody, everything, everywhere into a network. Of course the Internet has also changed how existing networks (e.g. transportation) behave.

This provides the motivation for this paper – networks in transportation. Part II reviews the mathematics or use of mathematics in transportation networks. Part III reviews the theory of network evolution and growth, and the economic theory in market with networks. In Part IV, whether networks in transportation create barriers to entry is briefly examined. Part V provides a few concluding remarks

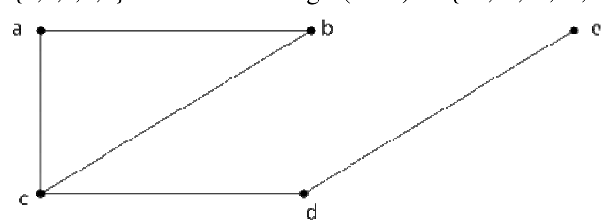
II. The mathematics or use of mathematics in transportation networks

In solving problems in transportation networks, graph theory in mathematics is a fundamental tool. The term 'graph' in mathematics has two different meaning. One is the graph of a function or the graph of a relation (eg. a stock price over time). The second, usually related to "graph theory", is a collection of 'vertices' or 'nodes' and, "links" or 'edges'. For purposes of this paper we are concerned with the latter type. Graph theory has been closely tied to its applications and its use first can be credited to transportation (Euler- the Konigsberg bridge

1736) followed by its application to other fields - electrical networks (Kirchhoff-1847), organic chemistry (Cayley-1857), and puzzles (Hamilton-1857). Its use today has spread and extended to many other fields of study (e.g. chemistry, computer science, ecology, genetics, physics, telecommunications, transportation networks).

In transportation, graph theory is most commonly used to study problems of: **A.** Routing - the one way street problem, the Konigsberg bridge problem, the Chinese postman problem, the travelling salesman problem, etc; and **B.** Networks - the maximum flow problem, the minimum cost flow problem, the transportation problem, etc. We shall describe these problems and show how graph theory is used to resolve them. But first a few terms shall be explained.

A graph 'G' in the above sense consists of two things: a set V whose elements are called 'vertices' or 'nodes'; and a set E of unordered pairs of distinct vertices, called 'edges'. It is commonly denoted as G(V, E). In the example of a simple graph G below, the set of vertices (points) V={a,b,c,d,e} and the set of edges(lines) E={ab,ac,bc,cd,de}.

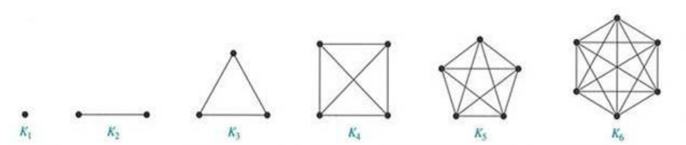


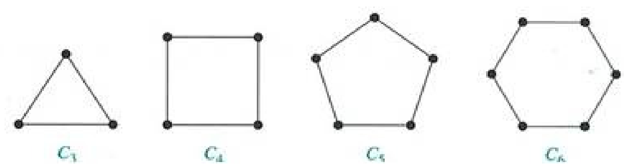
Graph G=(V,E) with Vertices V={a,b,c,d,e} and edges E={ab,ac,bc,cd,de}

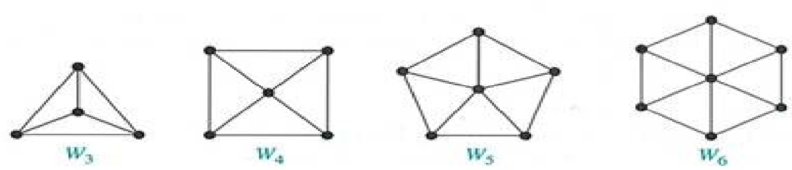
If the edges have a sense of direction then it is usually referred to as a directed graph(digraph). Sometimes multiple connections between the same vertices are allowed (multigraph) and sometimes loops (pseudograph). Two vertices are said to be adjacent (or neighbours) if there is an edge from one to the other. The degree of a vertex is the number of edges at that vertex. In the example above deg(a)=2, deg(b)=2, deg(c)=3, deg(d)=2, deg(e)=1. If we add up the degrees from a graph we get twice the number of edges because each edge gets counted twice, once for the vertex at either end.

There are some names used for special types of graphs, “null graphs” with no edges, “complete graphs” with every vertex joined to every other vertex, “cycles” which only join the outside of the vertices, “wheels” which add a vertex at the centre (see below from Rosen, p. 448).

Complete







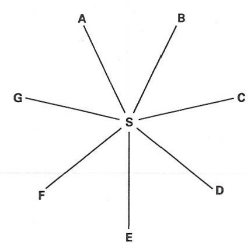


Figure 13.2
A star network

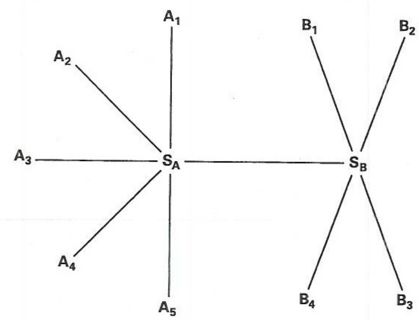


Figure 13.3
A long-distance network (alternatively, an ATM network)

(Economides, pp. 472-474.)

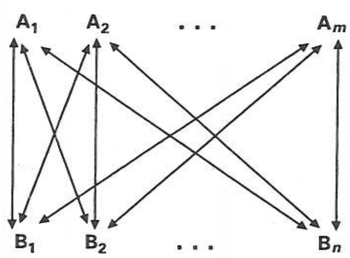


Figure 13.4
A virtual network of complementary goods

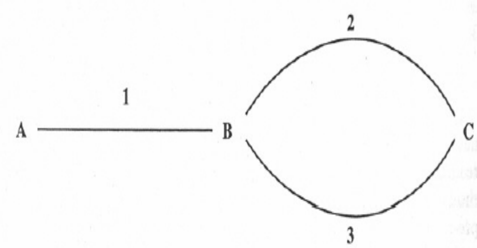


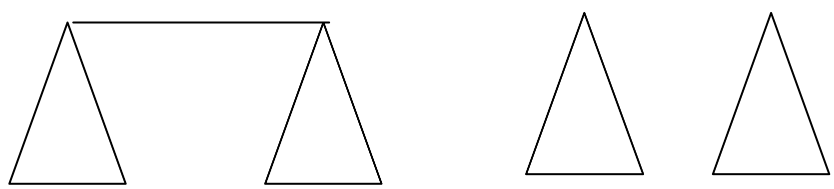
Figure 5.5 A one-sided bottleneck

(Economides, p. 110.)

A) Routing

i) The one way street problem - One of the problems faced by cities to control the flow of traffic is to ask whether it would be possible to convert two-way streets into one-way streets without getting into trouble i.e., ending up with some places that one can get into and never leave. That is for every pair of locations (x and y), is it possible to reach x from y and vice versa? Let us take a graph 'G' in the shape of a square with four sides, then let us give it a orientation (i.e., direction to each edge of G or diagraphs going clockwise) that is strongly connected [1]. In such a case, it is always possible to get from one vertex to another.

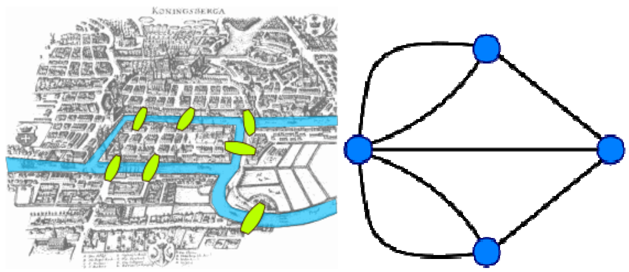
But for other graphs, this may not always be possible, for example take two triangles in one case with a bridge (i.e., a connecting edge) and in another case without a bridge.



(Roberts, p. 450)

To solve this problem one has to ensure that Robbins' theorem (which states: *A graph G has a strongly connected orientation if and only if G is connected and has no bridges*) is satisfied. But the theorem does not tell us how to find an assignment for a connected bridgeless graph. One way to do so is to use an algorithm known as *A Depth First Search Algorithm.* This algorithm, however, does not tell us whether it is efficient or inefficient in the sense of being the shortest one way street assignment. Fred Roberts states that there is evidence that no such algorithm exists.

ii) The Konigsberg bridge problem - The town of Konigsberg had seven bridges and its people wanted to know if one could start at some point, cross each bridge exactly once and return to the starting point. L. Euler whose name has been credited for solving this problem translated it into a graph theory problem.



The Königsberg Bridge problem, (Wikipedia – Graph Theory)

The problem can be solved if it has a eulerian closed chain. A chain or path is eulerian in a multigraph G if it uses every edge of G once and only once. This led to the formulation of Euler's famous theorem (*A multigraph G has an eulerian closed chain if and only if G is connected up to isolated vertices and every vertex of G has even degree).*

In other words, G must be connected up to isolated (has no neighbors) vertices, that is, at most one component has an edge. In addition, each vertex has an even degree (each edge must leave a vertex as often as it enters). Since the Konigsberg graph does not have a eulerian closed chain (some vertices are odd degree), the people of Konigsberg could not complete the walk as wanted. To solve problems for graphs that are not closed or multidiagraphs other conditions need to be satisfied (see other theorems by Euler and Good).[2] Variations have been added to the original problem by adding one, two and three more bridges or edges and inquiring whether the walk is possible and from which vertex.

iii) The Chinese postman problem - The Chinese postman problem or postman problem has been used to describe finding the shortest delivery or shortest carrier's route that involve starting from a point or post office in a territory and returning to the point or post office. Once again graph theory is used to solve the problem by building a graph G with each vertex representing a street corner and each edge representing a street. In other words, the mail carrier seeks a closed chain beginning and ending at the same point, using each edge at least once. If the graph has an eulerian chain, one can begin at a point and end at the same point. No chain can give a shorter route. If there is no such eulerian closed chain, modify G, by adding enough copies of each edge to exactly achieve the mail carrier's route. One may be able to do this in more than one way. The solution is to add the smallest number of copies of edges of G to obtain a multigraph which has a eulerian closed chain. In other words, solution to these problems (and others eg. street sweeping, RNA chains, etc.) require the application of a eulerian chain. Variations to the Chinese problem have been studied.

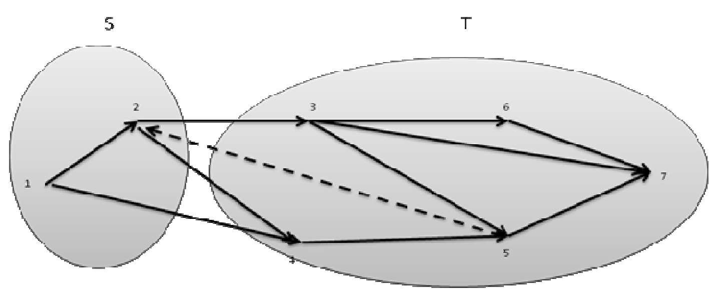
iv) The travelling salesman problem - The salesman problem is used to describe a situation when a salesman wishes to visit n different cities, each exactly once, and return to the starting point, in such a way as to minimize cost. Once again, graphs are used to formulate the problem where the cities are denoted by vertices or a complete symmetric diagraph. The arcs of the diagraph are given a weight, representing its costs (cost from one city to another). To solve this problem we seek a hamiltonian cycle in this diagraph which has minimum sum of weights. (A hamiltonian chain or path is one that uses each vertex once and only once). However, not every graph has a hamiltonian circuit. There are conditions for the existence of a hamiltonian circuit and cycle in a graph and diagraph. These conditions are expressed in theorems by Ore, Dirac, Bondy and Chvatal, Woodall, Ghouila-Houri. In other words, solution to these problems (and others eg., tournaments, etc.) require the application of a hamiltonian chain.

v) The shortest route problem - The shortest route problem is finding the shortest route between two vertices or cities represented by them in a network. To solve this problem we use a directed graph network or directed network and place weights indicating length on each arc. If it is a small network with a few arcs one can manually calculate the shortest route and there may be more such routes. But in a large network, finding the shortest route may be very tedious. To avoid doing it manually, an algorithm known as Dijkstra's Algorithm is used on a directed network. This is sometimes called the single-pair shortest path problem to distinguish it from other problems i.e., single-source shortest path problem, single destination shortest path problem and all-pairs shortest path problem. Other algorithms exist for solving these problems (or when weights are negative) such as Bellman-Ford, A* search, Floyd-Warshall, and Johnson's algorithm. Additional algorithms and associated evaluations may be found in Cherkassky et al.”[3]

B) Networks

The Concise Oxford dictionary defines it as an Aarrangement with intersecting lines & interstices... In mathematical topology, it is defined as ...a figure (in a plane or in space) consisting of a finite, non-zero, number of arcs, no two of which intersect except possibly at their end.[4] In this sense, it is more restrictive than some of the graphs described earlier.

i) The maximum flow problem - The maximum flow problem is to find a feasible flow through a single-source, single-sink flow network that is the maximum. The maximum value in a directed network of an s-t flow is equal to the minimum capacity of an s-t cut in the network, as stated in the max-flow min-cut theorem. (A cut can be considered as the set C of all arcs that go from vertices in set S to vertices in set T eg. If S={1,2} and T={3,4,5,6,7} then C={(1,4), (2,3), (2,4)} note (5,2) is excluded as it goes from set T to set S).



An (s, t) flow is maximum if and only if it admits no augmenting chain from s to t. An augmenting chain C is one in which the flow (x_{ij}) is less than the weight or capacity of the arc (c_{ij}) for each forward arc and the flow (x_{ij}) is greater than 0 for each backward arc. The above theorem is based on the assumption that there exists a maximum flow. If some capacities are not rational numbers, the maximum still exists, though the algorithms for solving it do not necessarily find a maximum. There are a number of algorithms for solving this problem. The best known are: the Ford-Fulkerson; Edmond-Karps; and Dinitz Blocking Flow.[5] Besides the above problem, there are a number of interesting applications that follow from the integral flow theorem which could have applications to problems in transportation. These are: Multi-source multi-sink maximum flow problem; Minimum path cover in directed acyclic graph; Maximum cardinality bipartite matching; Maximum flow problem with vertex capacities; Maximum independent path; and Maximum edge-disjoint path (Wikipedia).

ii) The minimum cost flow problem – The minimum cost flow problem is finding the cheapest possible way of sending a certain amount of flow (positive) from source s to sink t through the network at a minimum cost (i.e., nonnegative capacity (c_{ij}) on each arc and nonnegative cost (a_{ij})). The transportation problem can be viewed as a minimum cost flow problem. Imagine that a particular commodity is stored in n warehouses and is to be shipped to m markets. Let a_i and b_i be the supply and demand at each warehouse and market, respectively. Let a_{ij} be the cost of transportation and let total supply equal to total demand (i.e., $\sum a_i = \sum b_j$). The problem is to find a shipping pattern that minimizes the total transportation cost ($\sum a_{ij} x_{ij}$ i=1...n, j=1...m) subject to certain constraints --- total amount of commodity shipped from the i^{th} warehouse is at most the amount there ($\sum x_{ij} \leq a_i$, j=1...m) and total amount of commodity shipped to the j^{th} market is at least the amount demanded ($\sum x_{ij} \leq b_j$, i=1...n). Any solution satisfying these two constraints will also satisfy $\sum x_{ij} = a_i$, j=1...m; and $\sum x_{ij} = b_j$, i=1...n - since supply equals demand.

Thus, the solution to these problems requires certain assumptions to be satisfied. The problems can be resolved either using the simplexmethod or network flow theory. It is worthwhile noting that there is a difference between the transportation problem and transshipment problem. The latter permits shipping via intermediary nodes whereas the former does not. Algorithms have been developed to solve these problems (Wikipedia). More general problems are minimum cost maximum flow problem and minimum cost circulation problem.

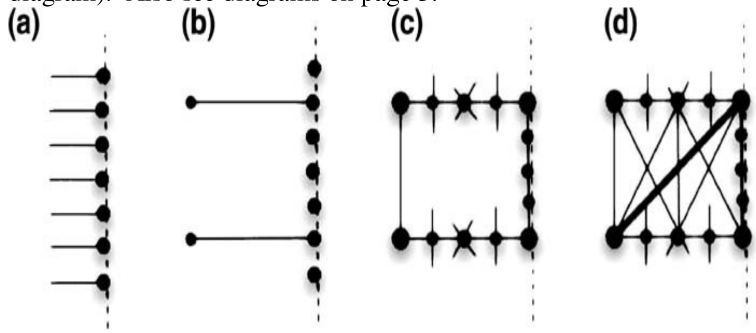
In sum, we have attempted to briefly describe the use of graph theory in mathematics to solve some of the basic network transportation problems. We have attempted to simplify it with the use of graphs and keep it as verbal as possible. Besides graph theory there are other types of mathematics that can be used to solve these problems such as combinatorics, game theory and linear programming techniques.

III. The Economic Theory of Networks in Transportation

The concept of network in economics is not new. It was implicit in the work of Quesnay (1758), Cournot (1838) and Pigou (1920). The latter studied a network system in the setting of a transportation network consisting of two routes and noted that the 'system-optimized' solution was distinct from the 'user-optimized' solution. More recently it originated in the work of Samuelson (1952), Takyama and Judge (1971), and Dafernos and Nagureny (1985) who identified the isomorphism between traffic network equilibrium problems and spatial equilibrium problems. Some writers have identified the general equilibrium problem known as Walrasian equilibrium as a network equilibrium problem over an abstract network.[6]

i) Theory of Network Formation and Growth in Transportation – Network theory has been developed in various branches of science, including economics and transportation. Its emphasis from time to time has shifted. A review of some of the literature and review articles does not indicate that there is any general theory of network formation in transportation. The theory of how and why networks have formed in one mode of transportation may not apply to another mode and going up and down the web of history does not provide a unique answer. Various attempts have been made and an excellent review (Xie and Levinson) on the subject indicates that studies have developed along three main streams: geography of transport networks; optimization and design of networks; and science of networks. Two other streams worthy of mention are: statistical analysis of networks; and economics of networks.[7]

Geography of transport networks (1960s): The geography of transport network models determine the transportation networks in terms of their structural formation/transformation and topological changes. It first introduced graph theory to model continuous growth of transportation networks beginning with: a few scattered nodes representing ports along the coast; a few roads penetrating inland from these ports; a few inland feeders and connecting ports because of trading; and links developed to interconnect developed nodes (see diagram). Also see diagrams on page 3.



(Xie and Levinsen, p. 293.)

Other studies, instead of following the above historical growth pattern, replicated observed networks attempting to simulate the changing topology. For example, in rail, the first link is added to connect the two largest settlements and then links are gradually added or as a tree branching out to connect outlying peripheral nodes. This first stream of network studies became dormant for thirty years as they were criticized for failing to emphasize the mechanisms as to why and how networks form and grow.

Optimization and design of networks (1970s): The optimization network models determine cost minimization of building and maintaining a network or the optimal changes in transport supply (existing links or on networks) that minimize user costs on the network under budgetary constraints. See examples on pages 5 and 6. The optimization models were developed with the availability of traffic flows and demand forecasting. Since then the concept of a bi-level optimal network design has dominated consideration of the evolution of networks within urban transportation planning. The lower-level represents the demand-performance equilibrium for a given investment while the upper level represents the investment decision-making of the transport planner to maximize social welfare based on the unique equilibrium flow patterns from the lower-level problem. This second stream has been subject to two main criticisms: it neglects the continuous interplay between many factors in shaping the structure of transportation networks; and lack of empirical evidence to show that transportation networks over time actually follow an optimal design.

Science of networks (2000s): The science of networks examines complex networks. It is based on the observation of a unique power-law distribution of a variety of 'scale-free' networks. As new nodes enter a free-scale network, they are more likely to get connected to highly connected nodes than less connected nodes, a process called 'preferential treatment'. In transportation, particularly surface, spatial constraints, limit the relevance and applicability of this growth mechanism of preferential attachment. It may, however, provide insights into: hub and spoke networks (non-geographical air transportation networks); independent node connection to established and important nodes; and self-organization and order into large scale networks. Recent work has concentrated on self-organization in complex systems and agent based simulation to interpret dynamics of transportation networks. This third stream is subject to the criticism that it may not apply to most transportation networks.

Statistical analysis of networks (1975-): The statistical analysis of networks investigates the temporal change of transportation supply based on historical observations. The statistical analysis grew with the availability of appropriate types of data. They relate the change of transportation supply to the demographic and socio-economic characteristics of tributary areas, as well as traffic conditions and other attributes of infrastructure. Simultaneous equations were used to examine mutual causality of demand and supply (transit, roads, infrastructure, etc.).

Economics of networks (1990s): The economics of network growth examines various economic dimensions of network growth, ranging from traditional transportation economics to public finance, path dependence, network effects, and coalition formation network. Transportation economics at the micro level examines network pricing, ownership structures and capacity investment and at the macro level examines the development of all modes of transport as a mix of private and government initiatives. Public finance relates to the provision of networks as a public good. Path dependence (present network system depends on the past) indicates that it may lead to lock-ins and market failure that are regrettable and difficult to change even in a world characterized by maximizing behaviour. Coalition formation models networks as the formation of links. They have developed as: strategic interaction models, random models and game theoretic models. The economic stream is theoretical in approach and were developed more to explain networks in other markets. Network effects examine the externalities (positive and negative) that result due to the demand and supply of networks. We shall examine coalition models and the network effects in greater detail hereafter.

ii) Strategic interaction models: The best known of these models is the connections model. This model (Jackson and Wolinsky)[8] simply states that people benefit from being well connected to each other. In this model, the payoff to agent i in a network g is

$$u_i(g) = \sum_{j \neq i} \partial l(ij) - d_i c$$

where $l(ij)$ is the number of links in the shortest path between i and j in g (setting $l(ij)=\infty$ if there is no path between i and j), d_i is i's degree (the links that i maintains in g), and c is a parameter representing the cost of a link. So people get benefits from maintaining direct connections that link one to another and also indirect benefits. The stability/equilibrium concept incorporates the idea that mutual consent is needed to form a relationship but people can unilaterally sever a relationship. "A network g is pairwise stable if: (i) if there is an i and ij not in g such that $u_i(g+ij)>u_i(g)$ then $u_j(g+ij)<u_j g$; and (ii)

for all ij in g and i $u_i(g) \geq u_i(g\text{-}ij)$'. If costs are low and below benefits links are formed and are pairwise stable. If costs are prohibitively high no links are formed. In cases when the costs are low or high, then the total utility maximizing networks is the unique pair wise stable network. When costs are intermediate, then the total utility maximizing network is a star, but stars[9] will often not be pair wise stable even when they maximize total utility. Other considerations have resulted in this basic model, being extended to include them. Examples of models are: spatial connections model, free trade models, market sharing models, social markets model, co-author model, etc.

b) The random models: Random models of network formation are from the random-graphs literature mainly developed by mathematicians and physicists. In these models the reason why a link is formed is pure chance. This literature builds networks either through a purely stochastic process where links appear at random according to some distribution or through some algorithm for building links. This approach is mainly dynamic and provides insight into how networks form.

c) The game theoretic models: Game theoretic models of network formation provide novel insight into the patterns that might emerge and into the tensions between individual incentives to maintain relationships and overall welfare. It takes the network structure as given and studies how the network structure impacts on outcomes and individual decisions. There can be multiple equilibria and such models can be difficult to solve.

iii) Network Externalities in Transportation - The theory of network externalities advances the notion that benefits or costs may arise on the supply-side or the demand-side that are not taken into account in the pricing mechanism. On the supply-side, the joint provision of service by members may result in economies or diseconomies that are not captured or paid for by providers of the network. Similarly, on the demand-side, externalities may arise because they are not captured or paid for by users of the network. These externalities arise because of the subtle interdependencies in the welfare of different units - interdependencies which cannot readily be reflected in the pricing arrangement.[10] This is because it cannot be easily measured or because mechanisms do not exist to collect them or to collect them efficiently. This has implications for the allocation of resources even in perfectly competitive markets.[11] Network effects have largely been developed in telecommunications (with regard to benefits) and transport (with regard to costs). These externalities provide the basis for the theory.

Examples of externalities can be found in transportation. On the supply-side, the establishment of a shipping line at a port may lead to expenditures to create a pool of related services which a new shipping line does not have to pay for. Or the expansion of an industry or a shipping line may make it cheaper for other shipping lines to operate because of lower cost in the supply of inputs. On the demand-side, the increase in cars or trucks on a highway network can increase congestion and ultimately result in gridlock which increases the cost to all vehicles which is not taken into account in the consumption or production pricing of transport services. Or the increase can result in more noise or air pollution whose costs do not have to be borne by the users of the highway. Or the addition of segments or arteries to a rail or road network may provide benefits to the existing users of these networks by enabling them to send traffic or travel to destinations that were previously not available to the existing users or by making complementary products available (eg. more fuel and restaurant facilities). In addition, the Internet allows the creation of all kinds of new products complementary to transportation.

The special features of markets with network effects have been described by Professor Economides. Those applicable to transport are highlighted. **First**, a firm can make money from either side of the network. **Second,** an additional user of the network is not rewarded for the benefit it brings to others. **Third**, the pace of market penetration (market expansion) is much faster in network industries than in non-network industries. **Fourth**, markets with strong network effects where firms can chose their own technical standards [eg. different rail gauges] are ‘winner-take-most’ markets resulting in extreme market share and profit inequality. Fifth, in industries with significant network externalities, under conditions of incompatibility between competing platforms, monopoly may maximize social surplus. **Sixth**, inequality is natural in the market structure of network industries. **Seventh**, free entry in network industries does not lead to perfect competition and eliminating barriers may not significantly affect market structure. **Eight**, 'winner takes most' is the natural equilibrium in these markets. **Ninth**, competition *for the market* takes precedence over competition *in the market* at least initially. **Tenth**, is the importance of path dependence - today's sales depend on past number sold.

In sum, studies to model the evolution and growth of transportation networks have ranged from geographical studies that aim to replicate geometries based on intuitive and heuristic rules, optimization studies that predict optimal network designs subject to an explicit objective function, to simulation studies that model network formation employing agent based methods. Economic studies to model transportation networks concentrate on strategic interaction models, random models and game theoretic models, their focus is more on networks and network effects than on transportation. So to-date there is no general economic theory on why and how networks are formed that is empirically verifiable.

IV. Are Networks in Industries Barriers to Entry

Barriers to entry arise from: absolute cost advantages over potential entrants; product differentiation advantages over potential entrants; and economies of large scale. By absolute cost advantages we mean established firms have no cost advantages in the purchase of factors of production; the entrant would have no perceptible effect on the going level of any factor price; the established firm has no preferred access to productive techniques. They arise from: control of production techniques (patents, IP); imperfections in the market or ownership or control of strategic factors; limitations of the suppliers of productive factors in specific markets; money market conditions (higher interest rates for potential entrants); and sunk costs. By product differentiation advantages we mean the preferences that buyers have for established networks over new networks. They arise from: brand names and company reputation; superior product designs through control of patents; and ownership or control of favoured distributive outlets. By economies of large scale we mean declines in cost as output increases. They arise from: real economies of large scale production or distribution (arising from indivisibilities of factors of production); pecuniary economies (bargaining) of large scale production; pecuniary economies of large scale advertising or sales.[12]

Do Transportation Network Industries have Absolute Cost Advantages? Being spatial and geographic established transportation network industries have an absolute cost advantage, some modes more than others. Land is a scarce resource in major cities, as a result, building new railway tracks or an airport with runways and buildings or a new sea port terminal is usually not possible. The price of land would certainly increase if there was any knowledge that a firm intended to build. Further, most of such projects need regulatory approval not only from transportation agencies but also from environmental agencies which usually takes many years. Furthermore, if there are any restrictions in the area of operation of an entrant as a result of licences or bilateral agreements there may be other absolute cost advantages. In addition most of these costs are sunk. In contrast to the above, in truck transportation or passenger bus transportation, the setting up of terminals may be much easier. This makes market penetration by a new entrant in most transportation network industries much slower than would occur in non-network industries.

Do Transportation Network Industries have Product Differentiation Advantages? Most established transportation network industries have product differentiation advantages arising out of company reputation and favoured locational advantages such as being down town or near the city. Their company reputation arises largely from their experience and number of years of being in business.

Do Transportation Network Industries have Economies of Large Scale? Transportation network industries do not have economies of scale in general, except the rail industry. However, they do have network economies, for example an additional user of the network brings benefits to other users of the network (through increases in load factor or traffic density thereby lowering cost and price), the pace of market penetration or expansion is much faster in network industries than in non-network industries. The existence of pecuniary economies, if any, is not considered to be significant.

In sum, we believe that there are barriers to entry in network transportation industries that range from high to medium to low depending on the mode and the specifics of each situation. Network effects have resulted in a market structure dominated by a few firms where bigger is better, in some modes of transportation.[13]

V. Concluding Remarks

Networks have always been important in transportation. They are widespread in Canada providing everyday links from home to work, life lines to diverse regions together with corridors for our export and import trade. Our economic futures are more closely tied to the sustainablility of our transportation networks than we might care to admit. They define the key to success in today's dynamic marketplace. They are a strategic tool to gain competitive advantage. This is particularly true today in the context of globalization.

The mathematical tools required to describe transportation networks is straightforward ranging from graph theory to linear programming. It enables one to logically formulate problems and solve problems related to routing - the one way street problem, the shortest route, the least cost route, etc. and problems related to network - the maximum flow, the minimum cost flow, the transportation problem, etc.

Network theory enables one to understand the formation, evolution and growth of networks in transportation. This improved understanding could reveal how decisions made in one point of time affect future choice. It could help planners and decision-makers desiring to shape the future by improving planning and designing of transportation networks so as to exploit network economies and externalities. It could help create optimal networks that will not only improve the flow of traffic but also reduce time lost due to congestion. It could also give planners an insight when certain segments are reaching capacity, so that they may plan alternative routes to avoid bottlenecks that could arise from sudden increases in both its own network traffic and traffic from other modes.

Networks in transportation sometimes create barriers to entry into the industry and at times it could create concern from the competition policy perspective. The reason is that theory indicates that such industries are likely to be dominated by a few large firms where winner takes most. This makes market penetration by a new entrant much slower than would occur in non-network industries and increases the likely duration and the value of market power. It thus increases the incentive and the likelihood that the dominant carrier will engage in anti-competitive acts to maintain its dominance or for a new carrier to obtain its market power. The anti-competitive acts can range from: denying access, tying, entering exclusive agreements, refusing to supply, etc. However, both the US and the EEC antitrust authorities have not shown any indication that network industries are so special as to require different treatment from other industries with essential facilities or that they need the application of a special set of rules in reviewing antitrust concerns at the present time.

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Endnotes

[1] To show that a graph is connected one usually employs a highly efficient procedure known as the *depth-first search procedure*. Efficiency here refers to computational complexity.

[2] See Fred Roberts, *Applied Combinatorics*, Rutgers University, 1984, pp. 460-461.

[3] See Wikipedia, **www.wikipedia.org**

[4] Arnold, B.H., *Intuitive Concepts in Elementary Topology*, 1963, p. 31.

[5] The others are listed in maximum flow problems in **www.wikipedia.org**

[6] Network Economics: An Introduction, Anna Nagurney, 2002.

[7] Feng Xie and David Levinson, Topological evolution of surface transportation networks, *Computers Environment and Urban Systems*, Volume 33, 2009, p. 211; and

Modeling the Growth of Transportation networks: A Comprehensive Review, *New Spatial Economics*, Volume 9, 2009, pp. 291-307.

[8] There are various types of networks: star (all nodes linked to centre node or hub and spoke); star with two dominant hubs; star with all nodes connected; empty networks (no nodes connected); networks with one or some nodes fully connected (referred to dominant group networks); generalized interlinked star (i.e., where the centre and three nodes are all connected and two nodes are only connected to the centre node).

[9] See reference 16 in Bibliography.

[10] Baumol, W., J., *Economic Theory and Operation Analysis*, p. 370.

[11] An interesting implication of this for competition advocates is that where there are external diseconomies, the presence of monopolies can lead to outputs smaller, and therefore more nearly optimal, than those which would result from competition. See Baumol, W., p. 370.

[12] Bain, J., *Industrial Organization*; and *Barriers to Entry*.

[13] In Canada, in rail and air transportation we have a duopoly and on some segments there is a monopoly.