SOME PROPERTIES OF REVERSE E-AUCTIONS WITH PRIVATE VALUES FOR THE THAILAND DEPARTMENT OF HIGHWAYS

Ponlathep Lertworawanich, Ph.D.

Bureau of Road Research and Development
Department of Highways, Thailand
Tel: 01-925-1653
Email: ponlathep@psualum.com or ponla_le@doh.go.th

ABSTRACT

Nearly any buying and selling transaction can, in fact, be viewed as the result of an auction process. With the average budget of 40,000 million Bahts, the Department of Highways (DOH), which is the primary agency responsible for planning, designing, constructing, and maintaining national highways of Thailand, hosts more than 500 highway construction-related auctions each fiscal year. The most typical form of auction used by the DOH is the first-price sealed-bid reverse auction. Until recently, E-auction has been introduced to the DOH auction process. This study investigates effects of each type of auctions on payment mechanism and efficiency. With private values, it has been shown that E-auctions are weakly equivalent to second-price sealed-bid auctions.

1. INTRODUCTION

In 193 A.D., having killed the Emperor Pertinax, in a bold move the Praetorian Guard proceeded to sell off the entire Roman Empire by means of an auction. The winning bid was a promise of 25,000 sesterces per man to the Guard. The winning, Didius Julianus, was duly declared emperor but lasted for only two months before suffering from what is perhaps the earliest and most extreme instance of the “winner’s curse” – he was beheaded (Krishna 2002).

Auctions are not new. Nearly any buying and selling transaction can, in fact, be viewed as the result of an auction process. Most people know auctions involving art, livestock, or real estate, where an auctioneer is looking for the highest price he can get. It is also generally known that most government businesses especially constructions are contracted on the basis of reverse auction, in which the auctioneer is looking for the lowest price. The Department of Highways (DOH), which is the primary agency responsible for planning, designing, constructing, and maintaining national highways of Thailand, has been hosting numerous auctions for construction-related deals with private sectors in order to continuously achieve cost-saving and efficient operations. Therefore, it is extremely important for the DOH to ensure that all deals and projects are properly auctioned off in the following aspects:

- *Payment*: the DOH is looking for the auction that will yield the minimum payment for the construction project auctioned off.
- *Efficiency*: an auction is successful if the bidder that values the construction project least ex post actually gets it. When the sales involves future delivery of services, as in many procurement auctions, efficiency means that contract is more likely to be carried out and the service provided at a high level.
- *Simplicity*: one of the objectives of auctioneers in most auctions is to get as many participants as possible. Keeping the rule simple.

The objectives of this paper are:

i. to provide an overview on common auction formats,
ii. to investigate effects of auctions on expected payments of auctioneers, and
iii. to evaluate efficiency of auctions and their optimal reserve prices.

The next section provides an outline of typical auction formats and details of mathematical derivations of the expected payments, optimal bidding strategies and their optimal reserve prices with some assumptions on private value distributions. Finally, conclusions and discussions are drawn from the research results followed by the recommended future research.

2. REVERSE AUCTIONS WITH PRIVATE VALUE AND THEIR PROPERTIES

This part consists of two sections. The first section briefly explains an outline of auction types and their related properties such as winner determination and payment mechanism as well as the weak equivalency between the second-price sealed-bid reverse auction and the so-called E-auction, which is recently brought in function by the DOH. The second section discusses basic assumptions of the first-price and second-price sealed-bid reverse auctions with private value. The mathematical derivations of their crucial properties namely, 1) expected payments, 2) bidding strategies, and 3) optimal reserve prices are provided in this section. The following paragraphs present details of each.

2.1 Auction Formats and Equivalent Auctions

As a principle government agency, the DOH must guarantee that every cent of the budget is spent in the most economical way. With the average budget of 40,000 million Bahts, the DOH hosts more than 500 highway construction-related auctions each fiscal year. The most typical form of auction used by the DOH is the first-price sealed-bid reverse auction, in which the project is awarded to the bidder with the lowest bid that does not exceed the reserve price. If the lowest bid exceeds the reserve price, the auction is reset.

Until recently, E-auction has been introduced to the DOH auction process. The process starts with identifying potential contractors who can offer high quality constructions. Then an online auction service conducts the process through a secure web site where an electronic solicitation is issued with unique and special requirements. Contractors or bidders are generally required to pre-register with the DOH and receive sufficient instruction on how to participate. All bidders must agree to abide by the terms of the auction service. The bidders are notified when the bidding period is open and they know the amount of time available for the process. Bidders submit their prices remotely from their offices. There is no communication or negotiation with bidders by the government, but the bidders can follow the process to see the status of their bid by watching the prices as they appear on the web site in real time. The DOH also watches the bidding process from the office. At the conclusion of the pre-established bidding time, a winner is selected. The project is awarded to the bidder with the lowest offer that does not exceed the reserve price. Apparently from the process, E-auction is an open auction because it formally requires bidders to be present digitally at the same time and place, offering bidders the opportunity to observe the behavior of other participants.

From auction theory perspective, E-auction is equivalent to second-price sealed-bid auction. In the second-price sealed-bid auction, each bidder submits a single bid independently and the project is awarded to the lowest bidder. Unlike the first-price sealed-bid auction, the price the auctioneer pays to the winner is the "***second lowest***" price. Even though this type of auction is rarely utilized in practice, it is of value to investigate. How the two auctions (the E-auction and the second-price sealed-bid auction) are equivalent?

In E-auctions, it makes sense for a bidder to remain in the auction until his value has been reached. He should not drop beforehand and certainly should not remain longer. Thus, the winning bidder will stay until the next-to-last-bidder drops out, meaning that he will receive the payment from the auctioneer in which the last bidder dropped out, or the second lowest bid (with the assumption that the bidding price is decreasing continuously with ***a small increment***). Therefore, E-auctions are equivalent to second-price sealed-bid auctions. This equivalence is weak because with E-auctions bidders obtain information throughout the process when they observe the price during the auction process and can adjust their estimates of project values accordingly. However, with private values – if each bidder knows the value of the project to himself at the time of bidding and even if he knows what other bidders willing to pay this does not affect his own valuation, the situation is called one of privately known values or private values – the equivalency still holds (Krishna 2002).

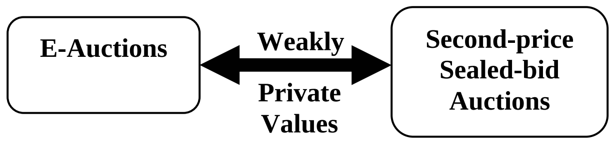


Figure 1: Equivalency of Auctions

2.2 Relevant Mathematical Derivation of Reverse Auctions with Private Values

In this section, basic properties of reverse auctions with private values and equilibrium bidding strategies are discussed. The assumptions of the models are stated as follows:

1. Private values: each bidder knows the value of the project to himself.
2. Independence: the values of different bidders are identically independently distributed (i.i.d.).
3. Risk neutrality: all bidders want to maximize their expected profits.
4. No budget constraint: all bidders have the ability to pay up to their values.
5. Symmetry: the values of all bidders are distributed according to same distribution function.

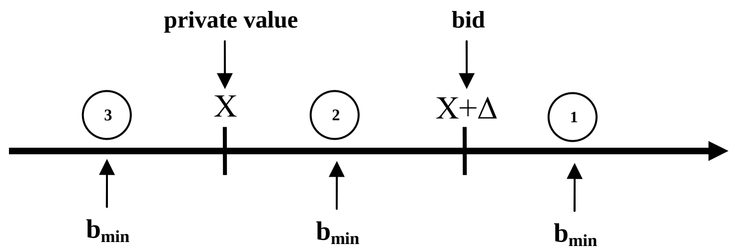
2.2.1 Second-price Sealed-Bid Auction (Equivalent to E-Auction)

Each bidder submits a sealed bid of b_i and given the private value of x_i, the expected payoff is:

$$\pi_i = \begin{cases} \min\limits_{j \neq i} b_j - x_i & \text{if } b_i < \min\limits_{j \neq i} b_j \\ 0 & \text{if } b_i > \min\limits_{j \neq i} b_j \end{cases}$$

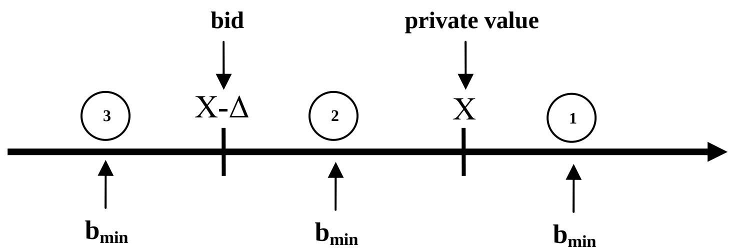
Lemma 1: In a second-price sealed-bid auction, it is weakly dominant strategy to bid according to $\beta^{II}(x) = x$ given the private value x. In other words, it is optimal to report the true value.

Proof:



Consider bidding $x + \Delta$: if the lowest bid other than yours is b_{min}, then the following may occur:

1. If $x + \Delta < b_{\min}$, you win (no different than bidding x).
2. If $x < b_{\min} < x + \Delta$, bidding $x + \Delta$ means you lose the auction, while bidding x would have meant winning with a surplus value of (b_{min}- x).
3. If $x > b_{\min}$, you lose (no different than bidding x).



Consider bidding $x-\Delta$: if the lowest bid other than yours is b_{min}, then the following may occur:

1. If $b_{\min} > x$, you win (no different than bidding x).
2. If $x-\Delta < b_{\min} < x$, bidding $x-\Delta$ means you win the auction but with a deficit of x-b_{min}, while bidding x would have meant losing but pay nothing.
3. If $b_{\min} < x-\Delta$, you lose (no different than bidding x).

It is obvious that bidding either ($x+\Delta$) or ($x-\Delta$) instead of x may only make payoff decrease. Therefore, it is the best strategy to bid one's own value in second-price sealed-bid auctions with private values.

Suppose that there are n bidders with private values $X_1, X_2, \ldots, X_n$ which are i.i.d. with a cumulative distribution function (CDF) of F(x) and a probability density function (PDF) of f(x). Denote $X_{(k)}$ the k^{th} lowest order statistic of $X_1, X_2, \ldots, X_n$. As a consequence, the ex ante expected payment by the auctioneer is:

$$\boxed{E\{X_{(2)}\} = \text{The expected value of the } 2^{nd} \text{ lowest order statistic of n private values}}$$

2.2.2 First-price Sealed-Bid Auction

Each bidder submits a sealed bid of b_i and given the private value of x_i, the expected payoff is:

$$\pi_i = \begin{cases} b_i - x_i & \text{if } b_i < \min\limits_{j \neq i} b_j \\ 0 & \text{if } b_i > \min\limits_{j \neq i} b_j \end{cases}$$

Bidder 1 wins the auction whenever he submits the lowest bid (b) or $\min\limits_{i \neq 1} \beta(x_i) > b$ where β is a bidding strategy function, which determines his bid for a given private value. Since β is an increasing function, $\min\limits_{i \neq 1} \beta(X_i) = \beta(\min\limits_{i \neq 1} X_i) = \beta(Y_1)$ where Y_1 is the lowest bid among the N-1 values. Bidder 1 wins whenever $\beta(Y_1) > b \Leftrightarrow Y_1 > \beta^{-1}(b)$. Therefore, his expected payoff is:

$$E(\Pi) = P[Y_1 > \beta^{-1}(b)] \cdot (b - x)$$

$$= \{1 - G(\beta^{-1}(b)\} \cdot (b - x) \qquad ; \quad 0 \leq x \leq b$$

At optimum, we can derive the equilibrium bidding strategy as follows:

$$\frac{dE(\Pi)}{db} = -\frac{g(\beta^{-1}(b))}{\beta'(\beta^{-1}(b))}(b - x) + [1 - G(\beta^{-1}(b))] = 0$$

At a symmetric equilibrium, b = β(x), $\beta(\omega) = \omega$

$$-g(x) \cdot [\beta(x) - x] + [1 - G(x)] \cdot \beta'(x) = 0$$

$$\beta^{I}(x) = \frac{1}{1-G(x)} \int_{x}^{\omega} y \cdot g(y) dy$$

where

$\beta^{I}(x)$ is the equilibrium bidding strategy for first - price sealed - bid auction

G(x) is the CDF of the Y_1, which is the lowest bid among N - 1 values

g(y) is the PDF of the Y_1, which is the lowest bid among N - 1 values

x,y are private values ranging $[0, \omega]$

As a result, the ex ante expected payment by the auctioneer is:

$$N \cdot E\{P[Y_1 > x] \times \beta^{I}(x)\} = N \cdot \int_{0}^{\omega}\int_{y}^{\omega} x \cdot g(x) \cdot dx \cdot f(y) \cdot dy$$

$$= \mathrm{N} \cdot \int_{0}^{\omega} x \cdot F(x) \cdot g(x) \cdot dx$$

$$= \mathrm{E}\{\mathrm{X}_{(2)}\}$$

$E\{X_{(2)}\}$ = The expected value of the 2^{nd} lowest order statistic of n private values

Interestingly, the ex ante expected payments by the auctioneers to the bidders for the first-price sealed-bid auctions are the same as that for the second-price sealed-bid auctions with private values.

Suppose that the private values are i.i.d. uniformly distributed [0,1] then the expected payment by the auctioneer is

$$E\{X_{(2)}\} = \frac{2}{N+1}$$

where

$E\{X_{(2)}\}$ is the ex ante expected payment by the auctioneer

N is the number of bidders

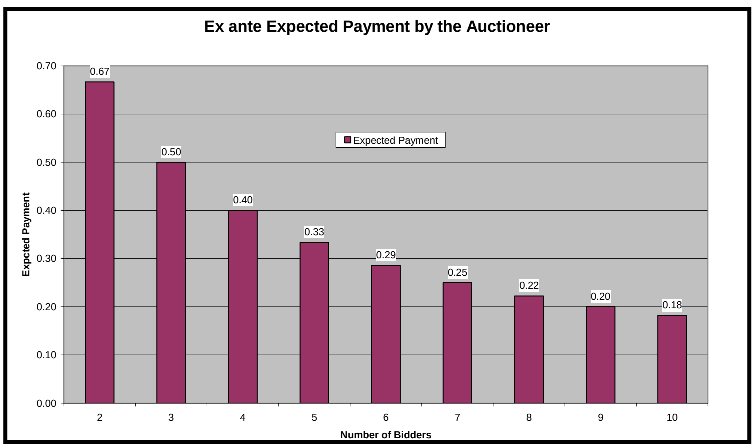


Figure 2: The Expected Payment by Auctioneer with Uniform [0, 1] Private Values

Referred to Figure 2, the expected payment by the auctioneer is decreasing with an increase in number of participating bidders.

2.2.3 Reserve Prices

In order to decrease payment, auctioneers should set a reserve price above which they should not auction off the project. This section investigates effects of reserve prices on the expected payment by the auctioneer.

a. Second-price Auction

Suppose that the auctioneer sets a reserve price r > 0. Since the price at which the project is auctioned off can never be higher than r, no bidder with x > r can make a positive profit in the auction. In a second-price auction, a reserve price makes no difference to the behavior of the bidders – it is still a dominant strategy to bid one's value. The expected payoff of a bidder with value r is just r[1-G(r)], and the expected payment of a bidder with value x ≤ r is:

$$m^{II}(x,r) = r[1-G(r)] + \int_{x}^{r} y \cdot g(y) \cdot dy$$

b. First-price Auction

In a first-price auction with a reserve price r > 0 since the price is at most r, no bidder with a value x > r can make a positive profit. If β^{I} is the symmetric equilibrium of the first-price auction with reserve price r, it must be that $\beta^{I}(r) = r.$ This is because a bidder with value r wins only if all other bidders have values greater than r and, in that case, he can win with a bid of r itself. In all other respects, the analysis of a first-price auction is unaffected. The bidding strategy is:

$$\beta^I(x) = E[\min\{Y_1, r\} \mid Y_1 > x]$$

$$= \frac{1}{1\text{-}G(x)}\{r \cdot [1 - G(r)] + \int_x^r y \cdot g(y) \cdot dy\}$$

The expected payoff of a bidder with value x ≤ r is:

$$\boxed{m^I(x, r) = r[1 - G(r)] + \int_x^r y \cdot g(y) \cdot dy}$$

Again, the expected payoffs of a bidder under the first or the second-price sealed-bid auction are alike even with a reserve price.

What is the optimal reserve price from the perspective of the auctioneer? Suppose that the auctioneer attaches a value $x_0 \in [0, \omega)$. This means that if the project is left unauctioned off, the auctioneer would have to pay only a value of x_0. Obviously, the auctioneer would not set a reserve price r that is greater than x_0. Then the overall expected payment of the auctioneer from setting a reserve price $r \le x_0$ is:

$$\Pi = N \cdot E\{m(X, r)\} + [1 - F(r)]^N \cdot x_0$$

In order to obtain the optimal reserve price, we take a derivative of Π with respect to r and equate it to zero.

$$\frac{d\Pi}{dr} = 0;$$

$$\{F(r^*) + r^* \cdot f(r^*)\}\{1 - G(r^*)\} = \{1 - F(r^*)\}^{N-1} f(r^*) \cdot x_0$$

3. CONCLUSIONS AND FUTURE RESEARCH

The study shows that there exists the weak equivalency between second-price sealed-bid reverse auctions and E-auctions with the assumption on symmetric private values. The findings illustrate the payment equivalence among first-price and second-price sealed-bid reverse auctions with private values for which the ex ante expected payment of the auctioneer is equal to the expected value of the 2^{nd} lowest order statistic among N- private values that are i.i.d and symmetric. An increase in number of participating bidders also helps decrease the expected payment of the auctioneer due to higher competition among bidders. In addition, it is optimal for bidders to submit one's true values in second-price sealed-bid auctions but this is not the case in first-price sealed bid auctions.

It is noteworthy that the findings in this study are not applied to the case of interdependent values among bidders for which the value of the items or projects auctioned is not known to the bidders but each bidder has only an estimate regarding the value or value is unknown at the time of the auction and may be affected by information available to other bidders.

In conclusion, the following model refinements and future research are recommended.

- Collect data and validate the proposed model,
- Develop an auction model that can account for interdependent values,
- Expand the methodology to evaluate effects of collusion among bidders,
- Disseminate research findings.

4. REFERENCES

[1]. Krishna, V. "Auction Theory" Academic Press, 2002.